MEMORANDUM FOR FILE

INTRODUCTION

The gravitational field of a planet may be estimated by reducing Doppler data between an orbiting satellite and an earth-based tracking station. This procedure has been used to determine the gravitational fields of the earth and moon. The gravitational parameter (μ) of the planets has been estimated from ephemeris data. The estimate of μ has been improved in the instance of Mars and Venus by reducing Doppler data from spacecraft which flew past the planets.

For a satellite orbiting Mars, the dominant errors in reducing Doppler data are in the size of the astronomical unit and the planet ephemeris. The procedure described in this work eliminates these errors by employing a Doppler link between two orbiting spacecraft. If the central body is non-spherical, a satellite's orbit plane will osculate with time. A new mathematical procedure is used to determine the gravitational field from the time varying orbits.

The technique described in this work was developed to accurately determine a planet's gravitational field. The procedure may also be used to estimate the gravitational field of the moon. This may permit determination of the gravitational anomalies on the backside of the moon.

MATHEMATICAL MODELS

A. Orbit Determination

The relative velocity between two points is expressed mathematically as the vector dot product $\dot{\rho} \cdot \frac{\rho}{|\rho|}$ where ρ is the vector between the points, $|\rho|$ is the Euclidean norm of ρ, and $\dot{\rho}$ is the velocity vector. The orbit determination problem is to determine a parameter vector X such that the least squares function $E = \sum_i \left(M_i - \dot{\rho}_i(X) \cdot \frac{\rho_i(X)}{|\rho_i(X)|} \right)^2$ is minimized, where M_i is measured velocity.

For an earth-based tracking station and a Mars orbiter, the vectors ρ and $\dot{\rho}$ are

$$\rho = r - R + D$$

$$\dot{\rho} = \dot{r} - \dot{R} + \dot{D}$$

where R = location of tracking station in an earth-centered frame,

$\dot{R}$ = velocity of tracking station in an earth-centered frame,

D = vector from center of Earth to center of Mars,

$\dot{D}$ = relative velocity between center of earth and center of Mars,

r = location of Mars orbiter in a Mars centered frame, and

$\dot{r}$ = velocity of Mars orbiter is a Mars centered frame.

The vectors R, $\dot{R}$, D and $\dot{D}$ are usually assumed to be known.

For two satellites orbiting Mars the vectors ρ and $\dot{\rho}$ are

$$\rho = r_1 - r_2$$

$$\dot{\rho} = \dot{r}_1 - \dot{r}_2$$

where

r_i = position of ith satellite in a Mars centered frame

$\dot{r}_i$ = velocity of ith satellite in a Mars centered frame

B. Time Independent Orbits

For time periods on the order of a day the perturbations on a high altitude satellite (periapsis altitude greater than 1000 km) produced by the non-spherical central body Mars are small. Thus, the motion of a satellite appears Keplerian.

The position and velocity of a satellite in a Keplerian orbit are

$$r = |r| \begin{bmatrix} \cos\Omega\cos(\omega+f) - \sin\Omega\sin(\omega+f)\cos i \\ \sin\Omega\cos(\omega+f) + \cos\Omega\sin(\omega+f)\cos i \\ \sin(\omega+f)\sin i \end{bmatrix} \tag{1}$$

$$\dot{r} = N \begin{bmatrix} -\cos\Omega(\sin(\omega+f) + e\sin\omega) - \sin\Omega\cos i\,(\cos(\omega+f) + e\cos\omega) \\ -\sin\Omega(\sin(\omega+f) + e\sin\omega) + \cos\Omega\cos i\,(\cos(\omega+f) + e\cos\omega) \\ (\cos(\omega+f) + e\cos\omega)\sin i \end{bmatrix} \tag{2}$$

where

$$|r| = \frac{a(1-e^2)}{1+e\cos f}$$

$$N = \left[\frac{\mu}{a(1-e^2)} \right]^{1/2}$$

a = semi-major axis

e = eccentricity

τ = time of periapsis passage

i = inclination

ω = argument of periapsis

Ω = longitude of ascending node

μ = gravitational parameter

The in-plane elements, a, e, and τ are independent of the coordinate system. The Euler angles i, ω, and Ω are referenced to the appropriate Mars centered coordinate system. The relationship between true anomaly f and time is given by Kepler's equation:

$$E - e\sin E = \sqrt{\frac{\mu}{a^3}}\,(t-\tau) \tag{3}$$

where

$\qquad$ E = eccentric anomaly

and

$$\sin f = \frac{\sqrt{1-e^2}\, \sin E}{1-e \cos E}$$

$$\cos f = \frac{\cos E - e}{1-e \cos E}$$

For earth-based tracking of a Mars satellite the orbit determination problem is to determine the vectors r and ṙ. For Keplerian motion this is equivalent to determining the parameter vector consisting of the six orbital elements a, e, τ, i, ω, and Ω plus the gravitational parameter μ.

With a Doppler link between two satellites, it is not possible to solve the orbit determination problem for twelve orbital elements, thereby describing the two orbits in some inertial reference frame. In this problem the matrix of second partial derivatives $\dfrac{\partial^2 E}{\partial X^2}$ is singular and there does not exist a unique solution to minimize the least squares error. The reason for this singularity is that the Doppler shift is determined by the relative orientation of the two satellites and there exist infinitely many sets of Euler angles yielding the same relative orientation.

However, the relative positions of two satellites can be determined with a Doppler link between the two satellites by using a Mars centered satellite coordinate system. In this system the fundamental reference plane is the satellite orbit plane of one satellite. The x axis points along the line of apsides towards periapsis. The z axis is parallel to and in the direction of the satellite angular momentum vector. The y axis completes a right hand orthogonal system. Thus, the satellite in the fundamental reference plane has zero for Euler angle values. In the Mars centered satellite coordinate system, the orbit determination problem is to determine a, e, and τ of the satellite in the reference plane, the six orbital elements of the other satellite and the gravitational parameter μ. This approach is equivalent to fixing the three Euler angles of one satellite and determining the nine remaining orbital elements and the gravitational parameter μ.

C. Time Varying Orbits

If the central body is non-spherical a satellite's orbit plane will osculate with time. The non-sphericity of a

planet can be considered in the orbit determination problem by
modifying the Keplerian analysis presented above. To illustrate
how the non-sphericity problem can be solved using a Doppler
link between two satellites, the central body will be assumed
to have an equatorial bulge.

General perturbation theory has shown that the oblate-
ness effects of a central body on a satellite can be expressed
by a time-varying state-vector of the satellite in terms of the
spherical harmonic components of the gravitational potential of
the central body. The perturbations can be grouped into (i)
secular variations, (ii) long-period variations, and (iii) short-
period variations. Kozai[1] has shown that for the first order
secular perturbation the short-period variations in a, e, τ, and
i can be considered as time independent and, for a coordinate
system having the equatorial plane as the fundamental reference
plane,

$$\Omega(t) = \Omega_o - \frac{3}{2} \frac{R_e^2}{a^2(1-e^2)^2} J_2 \bar{n}(\cos i)(t-t_o) = \Omega_o - \dot{\Omega}(t-t_o) \qquad (4)$$

$$\omega(t) = \omega_o + \frac{3}{4} \frac{R_e^2}{a^2(1-e^2)^2} J_2 \bar{n}(4-5\sin^2 i)(t-t_o) = \omega_o + \dot{\omega}(t-t_o) \qquad (5)$$

where

$$\bar{n} = \sqrt{\frac{\mu}{a^3}} \left[1 + \frac{3}{4} \frac{R_e^2}{a^2(1-e^2)^2} J_2 (2-3\sin^2 i) \sqrt{1-e^2} \right]$$

J_2 is the value of the second harmonic component the
central body potential.

R_e is the equatorial radius of the central body, and

Ω_o and ω_o are the initial values at $t=t_o$.

For a single satellite with earth-based tracking, the
J_2 term may be estimated by using the standard Mars centered
equatorial system, the time varying Euler angles $\Omega(t)$ and $\omega(t)$,
and by expanding the parameter vector X to consist of a, e, τ,
i, ω_o, Ω_o, μ and J_2. Such a procedure is identical to that used
for earth satellites.

For a Doppler link between two satellites estimating J_2 is mathematically more complex since relative orientations are determined using the Mars centered satellite coordinate system which is not referenced to the planet's equatorial plane. The equatorial bulge causes both satellite orbits to regress which produces time variations in all three Euler angles if neither satellite is orbiting in the equatorial plane. A technique to estimate the inclinations with respect to the equatorial plane and the J_2 term from the time variations in the Euler angles is described below.

In estimating the J_2 term a coordinate system having as its fundamental reference plane the equatorial plane is the easiest to use. Since the time variation of the Euler angles in such a system is independent of Ω_o and ω_o, a Mars centered satellite equatorial system will be used. In this system the fundamental reference plane is the Martian equatorial plane. The x axis points toward the ascending node of the satellite at time zero. The z axis points in the direction of the Martian north pole. The y axis completes an orthogonal right hand system.

Since two satellites are involved, satellite 1 will be taken as the reference orbit and its orbital elements denoted by a_1, e_1, τ_1, i_1, ω_1, and Ω_1. (Note that in the Mars centered satellite equatorial system Ω_1 is zero.) The orbital elements of satellite 2 will be denoted a_2, e_2, τ_2, i_2, ω_2, and Ω_2.

Introducing the time variation of the Euler angles in the satellite equatorial system produces Euler angles i_1, $\omega_1 + \dot{\omega}_1 \Delta t$, and $\dot{\Omega}_1 \Delta t$ for satellite 1 and Euler angles i_2, $\omega_2 + \dot{\omega}_2 \Delta t$, and $\Omega_2 + \dot{\Omega}_2 \Delta t$ for satellite 2. The corresponding Euler angles in the Mars centered satellite system will be denoted by i, ω, and Ω.

Let N_i be the normal to the orbit plane of satellite i and L_i a unit vector in the direction of periapsis of satellite i for $i = 1, 2$. In Appendix A it is shown that the following equations relate the various Euler angles,

$$N_1 \times N_2 \cdot L_1 - \cos \Omega = 0$$

$$N_1 \times N_2 \cdot L_2 - \cos \omega = 0 \tag{6}$$

$$N_1 \cdot N_2 - \cos i = 0.$$

The angles i, ω, Ω may be accurately estimated (see Table 2) using the Mars centered satellite coordinate system. Thus, Equations 6 are three equations in the unknowns i_1, ω_1, $\dot{\omega}_1$, Ω_1, $\dot{\Omega}_1$, i_2, ω_2, $\dot{\omega}_2$, Ω_2, and $\dot{\Omega}_2$. However, from Equation 5 it is seen that $\dot{\Omega}_j = \dfrac{2\cos i_j}{5\cos^2 i_j - 1}\dot{\omega}_j$ and that $\dot{\omega}_j$ is a function of J_2 and i_j for j = 1, 2. So the set of unknowns is reduced to the six unknowns i_1, ω_1, i_2, ω_2, Ω_2 and J_2. The angles i, ω, Ω are time dependent and given at least two distinct times, Equation 6 may be solved in the least squares sense for the six unknowns. Thus, not only may the J_2 term be estimated but also the Euler angles with respect to the Mars centered satellite equatorial coordinate system.

D.   Multiple Doppler Link

        Using a Doppler link between two satellites permits accurate estimation of their relative orientations. If the relative Euler angles are time dependent, the orientation with respect to the Mars centered satellite equatorial coordinate system may be determined. This reference system is dependent upon the orbit plane of satellite 1 and to determine the inertial orientation of the orbit plane of satellite 1 earth-based tracking must be used. The errors associated with earth-based tracking are discussed in the next section. By using the results obtained from the Doppler link between the two satellites only one parameter, namely Ω_1, need be determined from earth-based tracking.

The simulation shows that using two Doppler links permits a more accurate estimation of Ω_1 and of the other orbital elements than using only earth-based tracking. Appendix B describes the procedure for estimating the standard deviation of Ω_1 referenced to the standard Mars centered equatorial coordinate system using earth-based tracking and the orbital parameters determined in the Mars centered satellite equatorial coordinate system.

NUMERICAL SIMULATION

        In Reference 2 it was shown that the covariance matrix for the least squares error function yields an estimate of the standard deviations in estimating the orbital elements of a Mars satellite. Under the assumption that the standard deviation on each measurement is the same, the covariance matrix is of the form $\sigma^2\left[\dfrac{\partial F^T}{\partial X}\dfrac{\partial F}{\partial X}\right]^{-1}$ where F is the vector $(\dot{\rho}_j \cdot \rho_j/|\rho_j|)$ and σ is the standard deviation on each measurement. The square root of the diagonal elements is the estimate of the standard deviation for the respective components of the parameter vector X.

Thus, the attainable accuracies for earth-based tracking of either Mars satellite and for a Doppler link between two satellites may be determined by evaluating the respective covariance matrices and computing the standard deviations.

A.   Measurement Errors

In Reference 2 the following tabulation of velocity errors was presented:

Source	Value
Station Location	.00308 m/sec
Speed of Light	.000334 m/sec
Frequency Measurement	.0068 m/sec
Ephemeris	
Earth	.03 m/sec
Mars	.015 m/sec
A.U.	
Earth	.0199 m/sec
Mars	.0162 m/sec

To estimate the error in earth-based tracking the square root of the sum of squares of errors from all sources was taken. The resulting error estimate is .0423 m/sec. This compares with the 1σ error estimate of .03 m/sec, which did not include the A.U. error, used by JPL in processing Mariner IV data.

For a Doppler link between two satellites the dominant portion of the above estimate due to errors in station location, ephemeris and the A.U. is not present. The square root of the sum of squares of the errors in speed of light and frequency measurement is .00758 m/sec. Thus, the relative positions of two satellites using a Doppler link may be computed more accurately because of the smaller deviation than earth-based tracking for equal tracking times. This will be demonstrated by the results from a numerical simulation.

B.   Simulation Results

The orbits chosen for study were as follows. The two orbiting satellites have inclinations of 60° and 70°, periapsis altitude of 2090 km, and a period of twelve hours. There were 1440 data samples taken at one minute intervals, which is two orbits. For simplicity there was no Mars or Earth occultation included in the model.

Table 1 lists the orbital elements referenced to the Mars centered equatorial coordinate system and the standard deviations for earth-based tracking. Table 2 lists the orbital elements referenced to the Mars centered satellite coordinate system and the respective deviations on each measurement.

Table 3 shows the results obtained for J_2 and the
Euler angles referenced to the Mars centered satellite equatorial
system by solving Equation 6. As was noted earlier the relative
Euler angles are time dependent and a minimum of two distinct sets
of tracking data are required to estimate the Euler angles. The
two sets used were determined by using two separate days of track-
ing data, as described above, taken one week apart. The standard
deviation for the relative Euler angles used in computing the
entries in Table 3 was 1×10^{-5}. This value is an upper bound of
the standard deviations for the relative Euler angles in Table 2.
The estimation of the standard deviation of Ω_1, referenced to the
standard Mars centered equatorial coordinate systems, which is
determined from earth-based tracking of satellite 1, is also given.

Comparing Tables 1 and 2 it is seen that employing a
two way Doppler link permits more accurate determination of the
orbital elements than earth-based tracking. An increase in
accuracy of at least two orders of magnitude is expected because
of the decrease in standard deviation produced by eliminating
errors in station location, ephemeris, and A.U. There is an
additional improvement of two orders of magnitude in estimating
the relative Euler angles and the gravitational parameter. The
additional accuracies are a result of the geometry of two orbiting
satellites. Table 3 shows that the J_2 term may be estimated to
4 places and that the inclination, i, to the Mars equatorial
plane and the argument of periapsis, ω, may also be estimated
to 4 places by using the technique described earlier.

CONCLUSION

A Doppler link between two orbiting satellites provides
a means of accurately determining their relative positions. If
the planet is non-spherical, then the perturbation forces acting
on the satellites may be estimated and the orientation of the
satellite orbits with respect to the planet equatorial plane may
be determined. With this knowledge the orientation in inertial
space can be determined from earth-based tracking of either
satellite.

Simulation results show that the multiple link Doppler
system permits more accurate estimation of the orbital elements
than a single Doppler link between an earth-based tracking station
and an orbiting satellite. The numerical example presented should
be considered as a general order of magnitude estimate because the
standard deviations are functions of the geometry of the problem.
Only the J_2 term was considered but the procedure described can
be extended to include higher order spherical harmonic terms.

C. L. Greer

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Attachments

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The equations relating a planet equatorial coordinate system to the relative satellite coordinate system are derived below.

The position of a satellite in a Keplerian orbit is

$$\underline{r} = |\underline{r}| \begin{bmatrix} \cos \Omega \cos(\omega+f) - \sin \Omega \sin(\omega+f) \cos i \\ \sin \hat{\Omega} \cos(\omega+f) + \cos \Omega \sin(\omega+f) \cos i \\ \sin(\omega+f) \sin i \end{bmatrix} \qquad (A1)$$

where

$$|r| = \frac{a(1-e^2)}{1+e \cos f}$$

a = semi-major axis

e = eccentricity

τ = time of periapsis

i = inclination

ω = argument of periapsis

Ω = longitude of ascending node

μ = gravitational parameter

The orbital elements of satellite 1 will be subscripted by 1 and those of satellite 2 subscripted by 2. In the satellite coordinate system using the orbit plane of satellite 1 as the fundamental reference plane, the x axis points in the direction of periapsis. The z axis is in the direction and parallel to the angular momentum vector. The y axis completes a right hand orthogonal system.

The direction of periapsis, which corresponds to a true anomaly of zero, is the vector

$$L_1 = \begin{bmatrix} \cos \Omega_1 \cos \omega_1 - \sin \Omega_1 \sin \omega_1 \cos i_1 \\ \sin \Omega_1 \cos \omega_1 + \cos \Omega_1 \sin \omega_1 \cos i_1 \\ \sin \omega_1 \sin i_1 \end{bmatrix} \qquad (A2)$$

The normal to the satellite orbit plane may be computed by taking the vector cross product between the vectors whose true anomalies are $-\omega$ and $-\omega + 90°$. The normal N_1 is seen to be the vector

$$
N_1 = \begin{bmatrix} \sin i_1 \sin \Omega_1 \\ -\sin i_1 \cos \Omega_1 \\ \cos i_1 \end{bmatrix} \tag{A3}
$$

Proceeding as above gives

$$
L_2 = \begin{bmatrix} \cos \Omega_2 \cos \omega_2 - \sin \Omega_2 \sin \omega_2 \cos i_2 \\ \sin \Omega_2 \cos \omega_2 + \cos \Omega_2 \sin \omega_2 \cos i_2 \\ \sin \omega_2 \sin i_2 \end{bmatrix} \tag{A4}
$$

and

$$
N_2 = \begin{bmatrix} \sin i_2 \sin \Omega_2 \\ -\sin i_2 \cos \Omega_2 \\ \cos i_2 \end{bmatrix} \tag{A5}
$$

The vector $N_1 \times N_2$ is the line of intersection between the two orbit planes and is the line of nodes in the satellite coordinate system. In particular, $N_1 \times N_2$ points in the direction of the ascending node of satellite 2.

Thus the longitude of the ascending node satisfies

$$
\cos \Omega = N_1 \times N_2 \cdot L, \tag{A6}
$$

the argument of periapsis satisfies

$$
\cos \omega = N_1 \times N_2 \cdot L_2 \tag{A7}
$$

and the angle of inclination satisfies

$$
\cos i = N_1 \cdot N_2. \tag{A8}
$$

The geometry of the problem is illustrated in Figure A.

APPENDIX B

Let M_i be a vector of measured values which is to be approximated in the least squares sense by a vector function $F(i,\theta,Y)$. It is assumed that θ is a scalar variable and Y is a vector of constants with known standard deviations. The least squares error function is of the form $\sum\limits_{i} (M_i - F(i,\theta,Y))^2$. The value of θ which minimizes the least squares error satisfies the equation $\sum\limits_{i} \frac{\partial F(i,\theta,Y)}{\partial\theta} (M_i - F(i,\theta,Y)) = 0$. To first order the standard deviations satisfy the equation.

$$[\sum_{i} \frac{\partial^2 F(i,\theta,Y)}{\partial\theta^2} (M_i - F(i,\theta,Y)) + \sum_{i} \frac{\partial F(i,\theta,Y)}{\partial\theta}^2] \sigma_\theta^2 =$$

$$\sum_{i} \frac{\partial F(i,\theta,Y)}{\partial\theta}^2 \sigma_{M_i}^2 + \sum_{j} [\frac{\sum \partial^2 F(i,\theta,Y)}{\partial\theta\partial y_j} (M_i - F(i,\theta,Y))$$

$$+ \sum_{i} \frac{\partial F(i,\theta,Y)}{\partial\theta} \frac{\partial F(i,\theta,Y)}{\partial y_j}] \sigma_{Y_j}^2$$

It is assumed that σ_{M_i} is a constant σ for all i.

Neglecting second order terms and solving for σ_θ yields

$$\sigma_\theta = \sqrt{\frac{\sigma^2 \sum\limits_{i} \frac{\partial F(i,\theta,Y)}{\partial\theta}^2 + \sum\limits_{j} [\sum\limits_{i} \frac{\partial F(i,\theta,Y)}{\partial\theta} \frac{\partial F(i,\theta,Y)}{\partial y_j}]^2 \sigma_{Y_j}^2}{\sum\limits_{i} \frac{\partial F(i,\theta,Y)}{\partial\theta}^2}}$$

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REFERENCES

(1)   Kozai, Y., "The Motion of an Earth Satellite," Astronomical
      Journal, 64, 367-377 (1959).

(2)   Bayliss, S. S., "Error Analysis of Orbit Determination for
      Mars Orbiters," Technical Memorandum 68-1014-9, Jan. 23, 1969,
      Bellcomm, Inc.

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TABLE 1

Earth Based Tracking

Total Tracking Time 24 Hrs.
Sampling Rate is One Per Minute

Element	Value	1σ Deviation
a_1	12665.0 km	.211 + 1
e_1	.5682599	.800 − 6
τ_1	0.0 hrs	.360 − 5
i_1	60.0°	.386 − 1
ω_1	0.0°	.351 − 1
Ω_1	0.0°	.445 − 1
μ	5.5637 + 11 km^3/hr^2	.278 + 9
a_2	12665.0 km	.470 + 1
e_2	.5682599	.127 − 5
τ_2	1.0 hrs	.547 − 5
i_2	70.0°	.116 − 1
ω_2	140.0°	.234 − 1
Ω_2	70.0°	.36 − 1
μ	5.5637 + 11 km^3/hr^2	.619 + 9

TABLE 2

Two Satellite Doppler Link

Total Tracking Time 24 Hours
Sampling Rate is One Per Minute

Element	Value	1σ Deviation
a_1	12665.0 km	.579 − 3
e_1	.5682599	.206 − 7
τ_1	0.0 hrs	.885 − 7
a_2	12665.0 km	.579 − 3
e_2	.5682599	.237 − 7
τ_2	1.0 hrs	.915 − 7
i	97.79°	.449 − 5
ω	76.07°	.969 − 5
Ω	63.30°	.946 − 5
μ	$5.5637 + 11$ km^3/hr^2	.763 + 5

TABLE 3

Two Data Sets Determined

One Week Apart

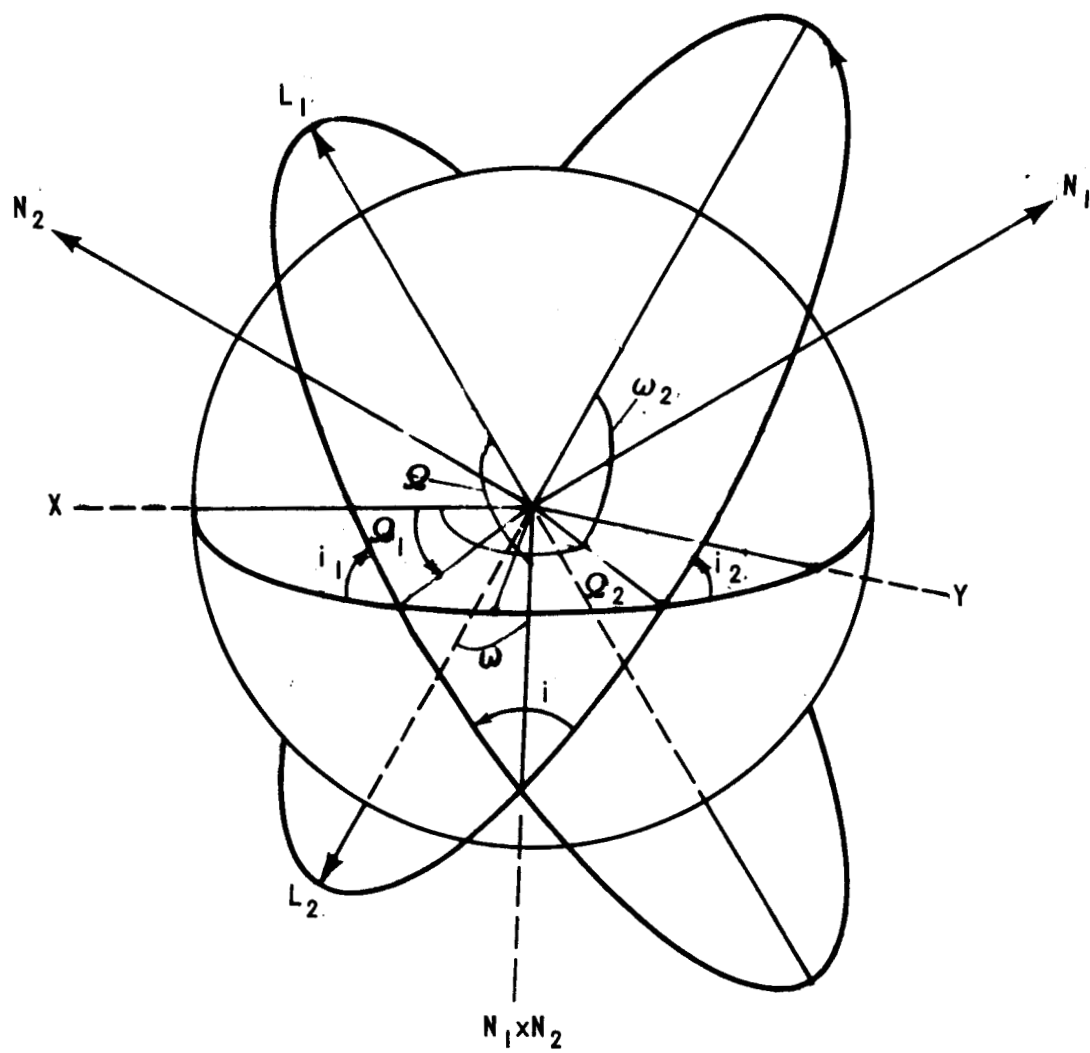| Element	Value	1σ Deviation
i_1	60.0°	.264 − 3
ω_1	0.0°	.678 − 4
i_2	70.0°	.100 − 3
ω_2	140.0°	.267 − 3
Ω_2	70.0°	.877 − 4
J_2	.00192	.514 − 6
Ω_1*	0.0°	.483 − 4

*Determined from Multiple Doppler Links

FIGURE A

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SUBJECT: Determination of Planet Gravitational Field Using Two Satellites Case 105-3

DATE: September 15, 1969

FROM: C. L. Greer

ABSTRACT

The gravitational field of a planet may be estimated using Doppler data from a radio link between an artificial satellite orbiting the planet and an earth-based station. When two satellites simultaneously orbit the same planet, as may well be the case with the 1971 Mariner spacecraft, the question occurs, can additional information and accuracy be obtained by providing a Doppler link between the two satellites.

Theory shows that satellite-to-satellite Doppler data, without the earth-based link, can be used to estimate relative satellite positions and the gravitational parameter, μ, for a symmetrical planet. For an oblate planet, this same data can be used to estimate satellite positions relative to the equatorial plane, as well as the degree of oblateness. Adding a Doppler link to earth allows the inertial coordinates of the satellite orbit planes to be included in the estimate.

To investigate the question of additional accuracy provided by the satellite-to-satellite Doppler data, a mathematical simulation of the data reduction problem was carried out on a digital computer. Under the particular assumptions employed, it was found that for equal tracking times estimated accuracies improved two to four orders of magnitude when the satellite-to-satellite data was used in conjunction with earth-based data, compared to earth-based data alone.